Int. J. Heat Mass Transfer. Vol. 36, No. 5, pp. 1133–1145, 1993
Printed in Great Britain

0017-9310/93 $6.00+0.00

Numerical prediction of periodically fully developed natural convection in a vertical channel with surface mounted heat generating blocks

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(*Received* 8 *July* 1991 *and in final form* 10 *June* 1992)

Abstract—Natural convection is often employed due to its simplicity and reliability in the cooling of electronic equipment operating at low heat fluxes. In closely packed arrays of circuit boards containing evenly spaced heated components, the buoyancy-induced flow becomes periodically fully developed rather quickly. The dependence of the characteristics of the fully developed natural convection flow on the type of imposed thermal boundary conditions is discussed. The mathematical formulation for periodically fully developed natural convection with thermally active throughflow is presented in the context of the flow through an insulated channel with heated blocks mounted on one of the walls. This represents a generalization of the theory of fully developed natural convection between parallel plates subjected to uniform heat flux that is available in the literature. The decomposition of the temperature field into a periodic part and a linearly varying part has implications in the discretization of the temperature field and is elaborated. The solution procedure for the discretization equations that addresses the constraint imposed on the periodic part of the temperature field is then described. Computations are carried out for the periodically fully developed natural convection in the channel geometry considered. The values of the channel length and module heating are varied to illustrate the utility of the proposed formulation and the solution procedure in analyzing a class of periodically fully developed natural convection flows encountered in electronics cooling applications.

INTRODUCTION

NATURAL convection is frequently employed in the cooling of electronic equipment. Although the heat removal capacity of natural convection flow is small in comparison to forced convection or immersion cooling, it is useful for cooling electronic equipment operating at low heat fluxes because of its inherent reliability and simplicity. In such applications, typically, the heat dissipating components are mounted on circuit boards that are arranged in vertically oriented arrays forming channels through which a flow of the coolant (typically air) is established due to buoyancy forces. The heated electronic components are usually arranged evenly on the circuit boards. This gives rise to a throughflow pattern, which after an initial adjustment, becomes periodically repetitive.

A large body of literature exists on studies of natural convection in open channels. A comprehensive review of theoretical and experimental work on the thermal control of electronic equipment that includes natural convection cooling has been made by Peterson and Ortega [1]. A number of investigations on natural convection in open channels have been performed. Theoretical studies of developing and fully developed natural convection between asymmetrically heated vertical flat plates have been carried out by Aung [2, 3] and by Aung *et al.* [4]. Experimental studies of natural convection in card arrays mounted in cabinets have been performed by Aung *et al.* [5, 6]. Wirtz and Stutzman [7] have performed measurements on free convection of air in symmetrically and uniformly heated channels. Sparrow *et al.* [8] have presented an experimental study of natural convection of water in an asymmetrically heated vertical channel which exhibits flow reversal. Numerical investigations of natural convection in in-line and staggered arrays of vertical isothermal plates have been conducted by Sparrow and Prakash [9, 10]. Bar-Cohen and Rohsenow [11] have presented composite correlations for natural convection heat transfer in heated channels and have described relations for obtaining optimum spacing between vertical plates to maximize heat transfer.

Extensive data on natural convection heat transfer from stacks of circuit boards with protruding components have been presented by Birnbreier [12]. Many

NOMENCLATURE

- A_b aspect ratio of the block
- A_m aspect ratio of the periodic module
- b channel width
- c_p specific heat of the fluid
- f force per unit volume of the fluid
- g gravitational acceleration
- G gap width parameter
- Gr_b Grashof number representing module heating
- h module-averaged heat transfer coefficient
- k_b thermal conductivity of the block material
- k_{eq} equivalent conductivity for a control volume face
- k_f thermal conductivity of the fluid
- K ratio of conductivities of the block and the fluid
- l length of the channel
- l_b length of the block
- l_m length of the periodic module
- $\dot{m}(\dot{M})$ dimensional (dimensionless) mass flow rate
- N number of modules in the channel
- Nu module averaged Nusselt number
- p static pressure
- p_0 static pressure at the channel entrance
- $\tilde{p}(x, y)$ periodic part of the pressure field
- $P(x, y)$ dimensionless periodic part of the pressure field
- Pr Prandtl number of the fluid
- q_x heat flow rate across a control volume face aligned in the x direction
- Q total heat generation within the periodic module per unit depth
- $r(y)$ quadratic part of the pressure field
- Ra_b Rayleigh number representing module heating
- T static temperature
- T_{block} uniform temperature attained by the block
- T_0 temperature of the environment
- $\bar{T}_b$ average of the inlet and the exit bulk temperatures for the periodic module
- $\tilde{T}$ periodic part of the static temperature
- $\tilde{T}_{b,0}$ bulk value of the periodic part of the static temperature at the entrance of the module
- $u(U)$ dimensional (dimensionless) velocity in Cartesian x direction
- $v(V)$ dimensional (dimensionless) velocity in Cartesian y direction
- $x(X)$ dimensional (dimensionless) distance in Cartesian x direction
- $y(Y)$ dimensional (dimensionless) distance in Cartesian y direction
- y_s y coordinate at the start of a module.

Greek symbols

- α_f diffusivity of the fluid
- α_m underrelaxation factor for mass flow rate
- β coefficient of thermal expansion of the fluid
- γ linear rate of rise of the bulk temperature
- δ iterative increment to the discretized temperature
- Δx, Δy dimensions of a control volume in the x and y Cartesian directions
- θ dimensionless periodic part of the temperature field
- ν dynamic viscosity
- ρ density of the fluid
- ρ_0 reference density of the fluid
- ψ dimensionless stream function.

Subscripts

- f fluid
- b bulk value, block.

Superscripts

- $+$, $-$ opposite sides of a control volume face.

studies of natural convection from discrete heated components mounted on vertical or inclined flat walls with and without shrouding surfaces are available in the literature. For example, Jaluria [13] and Jaluria and Gebhart [14] have performed theoretical and experimental studies to investigate natural convection flow due to isolated and interacting sources on adiabatic vertical surfaces. Natural convection from two interacting cylinders on a vertical surface has been presented by Milanez and Bergles [15]. Natural convection from discrete protruding sources on vertical surfaces has been studied almost exclusively using experimental techniques. Ortega and Moffat [16, 17] have performed experiments on natural convection from heated arrays of protruding cubical elements mounted on an insulated vertical flat plate with and without a shroud plate. The flow was found to be periodically fully developed within the first few rows of heaters for narrow channel spacings.

The focus of this study is to investigate periodically fully developed natural convection in channels which exhibit periodicity in geometry and in which multi-dimensional effects cannot be ignored. The effect of thermal boundary conditions on the nature of periodically fully developed natural convection is discussed. The mathematical formulation for the periodically fully developed natural convection with thermally active throughflow is presented for a particular physical situation. The discretization of the equations governing the periodically fully developed flow is then described with an emphasis on the discretization of the temperature field. A solution procedure that addresses the special features of this type of periodically fully developed flow is then outlined. Finally, the results of the computations for two-dimensional periodically fully developed natural convection in an insulated vertical channel with heated blocks mounted on one of the walls are presented.

PERIODICALLY FULLY DEVELOPED NATURAL CONVECTION

Periodically fully developed flow is attained in channels consisting of repetitive geometrical modules when the characteristics of the flow in successive modules become identical. In forced convection flows, the fluid is propelled through the channel by an externally imposed pressure difference. Therefore, periodically fully developed forced flow of a fluid through a channel with a specified geometry of the periodic module is governed only by the pressure drop imposed across a module. Natural convection flows, on the other hand, are caused by pressure imbalances arising out of the variation of the density of the fluid with temperature. Therefore, for a specified channel geometry, imposition of thermal boundary conditions that are different in character results in periodically fully developed natural convection flows which are qualitatively different in nature. Two types of periodically fully developed natural convection flows result depending upon whether there is a net transfer of heat across a module to the buoyant throughflow or not. The character of these two types of flows and the relationship with the corresponding thermal boundary conditions are now discussed.

Periodically fully developed natural convection with thermally inactive throughflow

This type of periodically fully developed natural convection flow results when a part or all of the bounding surfaces of the periodic module are maintained at or exposed to environments at specified temperatures. Under such conditions, the heat transferred into the module through the heated part of the module boundary or the heat generated within the module itself is transferred to the surroundings through the cooled part of the boundary across the throughflow. Hence the throughflow is termed thermally inactive and does not absorb any net heat as it traverses across a module. Then, in addition to the velocity field, the resulting periodic temperature field is also identical in all modules. When fully developed flow is attained, the induced mass flow rate is unaffected by any further increase in the number of periodic modules in the channel and it represents the asymptotic maximum mass flow rate that can be achieved by natural convection through the channel geometry with the prescribed thermal boundary conditions. The temperatures in the module in the fully developed regime also represent the limit of the maximum temperatures than can be reached in the channel.

Periodically fully developed natural convection with thermally active throughflow

When the boundaries of the periodic module in the channel are either subjected to specified heat flux, are insulated or are thermally rentrant, the heat generated within the module or the heat transferred into the module has to be absorbed by the throughflow. A periodically fully developed flow can be attained with these thermal boundary conditions provided the channel is sufficiently long. Unlike the previous situation, however, the bulk temperature of the induced flow rises continuously through the channel, the rise in the bulk temperature across each periodic module being the same. The buoyancy force, which induces the flow through the channel, is proportional to the average temperature in the channel. As the number of modules is increased, the average temperature in the channel and hence the buoyancy force increases, causing the induced flow rate to increase. Thus, the flow rate in the periodically fully developed regime is dependent on the number of modules in the channel and this behavior is fundamentally different from that observed in the fully developed flow with thermally inactive throughflow. This variation of the induced flow rate in the fully developed regime with the number of modules in the channel represents its limiting variation in a real channel as the number of modules becomes sufficiently large. Further, for a long channel, the developing flow in the first few modules has negligible effects on the characteristics of the overall flow.

In the draft cooling of modern electronic systems, circuit boards containing electronic components are typically arranged in a vertically positioned array which is usually enclosed in a cabinet [5, 6]. The buoyancy-induced flow through the array removes the heat that is generated by the electronic components. For densely populated, narrow channeled arrangements of the circuit boards, a periodically fully developed regime is attained within the first few modules in the channel. Since the circuit boards are arranged in an array form, the induced flow cannot transfer the heat to the surroundings and the temperature of the throughflow continues to increase as it traverses through the channel. Therefore, the character of this flow regime is akin to the type of periodically fully developed natural convection where there is net transfer of heat to the throughflow across a periodic module. The kind of periodic fully developed flow where the throughflow does not absorb any heat across a module is not very relevant to electronic cooling applications since attainment of such a regime assumes that the heated channels are isolated and directly exposed to the ambient and, hence, is not considered in the present study.

A theory of fully developed natural convection between vertical parallel plates subjected to uniform heat flux (UHF) has been presented by Aung [2]. The focus of this study is to extend the theory to multidimensional situations in which a periodically fully developed natural convection with thermally active throughflow is possible. In addition, a technique is devised for the numerical solution of the fully developed flow. The analysis of the fully developed regime yields useful information such as the induced mass flow rate and the maximum temperature. A parametric study of the physical situation under consideration can then be carried out to utilize such information in the design optimization of the coolant passages. In the discussion that follows, such an analysis of the periodically fully developed natural convection with active throughflow is carried out in the context of a particular physical situation. However, the mathematical formulation and the proposed solution technique are completely general and can be readily applied to analyze the entire class of physical situations in which periodically fully developed natural convection with thermally active throughflow is possible.

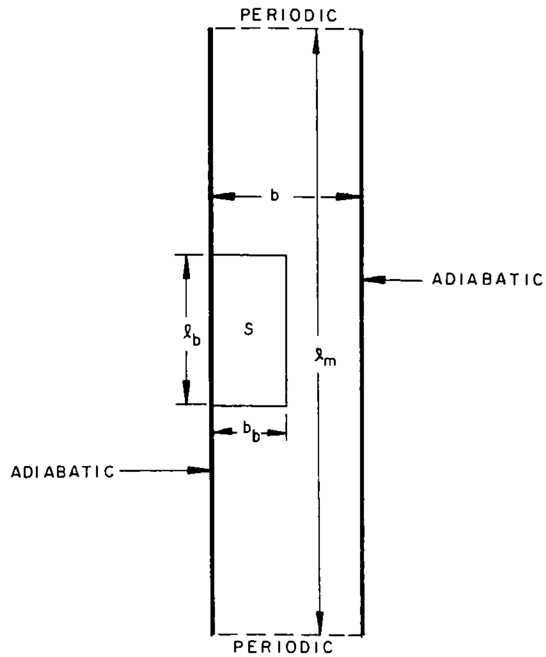

FIG. 2. Periodic module used for computations.

THE PHYSICAL SITUATION

The physical situation chosen for analysis in the present study is shown in Fig. 1. It is a two-dimensional idealization of a typical electronic circuit board array. Figure 2 shows the geometrical details and the boundary conditions for a periodic module in the channel. The heat generation within the block is assumed to be uniform and the conductivity of the block material can be different from that of the coolant fluid. The thickness of the boards on which the blocks are mounted is assumed to be small. The side walls of the channel can be assumed to be insulated or the temperature field can be assumed to be continuous across the side surfaces of the channel, implying a thermal periodicity or rentrant condition on the side walls. In the present study, the side walls are assumed to be insulated with a view towards illustrating the relevant aspect of the analysis of such periodically fully developed flows while keeping the problem definition simple. The channel is assumed to be oriented vertically so as to maximize the buoyancy-induced flow.

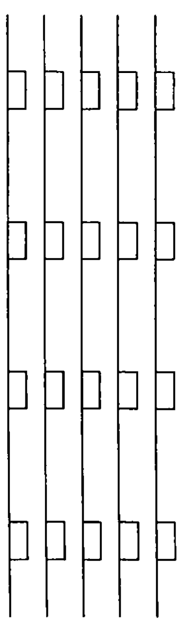

FIG. 1. An array of vertical plates with heat generating blocks.

MATHEMATICAL FORMULATION

Consider a laminar, steady flow of a Newtonian fluid with constant properties through the channel shown in Fig. 2. The equations governing the variation of velocity, pressure and temperature fields over a module in the periodically fully developed regime are now derived.

The induced mass flow rate and hence the characteristics of the periodically fully developed natural convection flow depend upon the number of modules in the channel for a specified module geometry and operating conditions. In the periodically fully developed regime of the flow in a channel with a fixed number of modules, the hydrodynamic and thermal characteristics of the flow are identical in each module. In particular, the velocity field in each module in the fully developed region is the same in all modules, and is subject to periodicity conditions. Since the throughflow absorbs the heat dissipated by the blocks, the temperature in the channel increases continuously and the temperature field by itself does not repeat periodically from module to module. However, the differences in temperatures at corresponding points in any two modules in the fully developed region are identical. In other words, the variation of the temperature relative to the average temperature in a module is identical in all modules. This variation of the temperature can then be decomposed into two parts in a manner similar to the decomposition of the pressure field in periodically fully developed forced flows as proposed by Patankar *et al.* [18]—a linear variation to account for the absorption of heat input Q in each module and a periodic part which is identical in all modules as follows:

$$T(x,y) = (\tilde{T}(x,y) - \tilde{T}_{b,0}) + \gamma y + T_0 \tag{1}$$

where $\gamma = Q/(\dot{m}c_p l_m)$, $\dot{m}$ being the induced mass flow rate.

Several points in the above expression are worth noting. First, the linear rate of rise of the temperature γ is dependent on the induced mass flow rate, which is not known *a priori* and is an outcome of the solution. Second, the temperature field $\tilde{T}(x,y)$ repeats periodically from module to module, $\tilde{T}_{b,0}$ being the bulk value of $\tilde{T}(x,y)$ at the entrance of the module. Further, the temperature field satisfies the condition that the bulk temperature at the entrance of the channel which corresponds to $y = 0$ is equal to the environment temperature T_0. Finally, the bulk temperature at the exit of the channel is given by

$$T_b|_{y=L} - T_0 = \frac{Ql}{\dot{m}c_p l_m} \text{ or } \frac{QN}{\dot{m}c_p} \tag{2}$$

where the length l of the channel is given by $l = Nl_m$, N being the number of modules in the channel. Thus, the temperature field as expressed in equation (1) automatically satisfies the overall energy balance for the channel assuming negligible viscous dissipation. The energy equation governing the temperature field can be recast in terms of the periodic part of the temperature $\tilde{T}(x,y)$ as the unknown and is described later in this section.

The buoyant flow through the channel is driven by an interplay of the pressure and the gravitational force on the fluid, which is dependent on the density and hence the temperature of the fluid. Since the buoyancy force by itself does not show periodic behavior due to the continuous increase in the temperature along the channel, the pressure field by itself cannot repeat periodically. In order for the velocity field to become periodically fully developed, the net result of the pressure and the body forces needs to be identical in each module and this condition is now utilized to determine the variation of pressure along the channel. Let

$$\vec{f} = -\vec{\nabla}p - \rho g\hat{j} \tag{3}$$

where the force per unit volume $\vec{f}$ needs to be periodic. Now, invoking the Boussineq approximation, the density can be expressed as follows:

$$\rho = \rho_0(1 - \beta(T - T_0)). \tag{4}$$

Further, following the theme used in the decomposition of the temperature field, the pressure field can be decomposed into a periodic part $\tilde{p}(x,y)$ and a portion $r(y)$ that depends on y as follows:

$$p(x,y) = \tilde{p}(x,y) + r(y). \tag{5}$$

Using equations (1), (3), (4), and (5) and imposing the periodicity condition on $\vec{f}$, an expression for $r(y)$ is obtained. Thus

$$r(y) = -\rho_0 g(1 + \beta\tilde{T}_{b,0})y + \rho_0 g\gamma\beta\frac{y^2}{2} + C. \tag{6}$$

Note that $\tilde{p}(x,y)$ can be viewed as a perturbation on the variation of the pressure $r(y)$ in the vertical direction due to the two-dimensional nature of the fully developed flow. Thus, the condition that the hydrostatic pressure at the inlet and the exit of the channel match the externally imposed pressure head can be imposed on $r(y)$ directly. Thus

$$r(0) = p_0 \quad \text{and} \quad r(l) = p_0 - \rho_0 gl \tag{7}$$

imply

$$r(y) = \rho_0 g(1 + \beta\tilde{T}_{b,0})y + \rho_0 g\gamma\beta\frac{y^2}{2} + p_0 \tag{8}$$

and

$$\tilde{T}_{b,0} = \frac{\gamma l}{2} \text{ or } \frac{QN}{2\dot{m}c_p}. \tag{9}$$

The significance of the condition on the bulk periodic temperature at the entrance of a module will become clear in the latter part of this section.

The Navier–Stokes equation can now be expressed in terms of the periodic velocity field and the periodic parts of the pressure and temperature fields to obtain the equations governing the periodically fully developed flow. The following definitions of the dimensionless variables are utilized in this study:

$$U = \frac{u}{\nu/b}, \quad V = \frac{v}{\nu/b}, \quad X = \frac{x}{b}, \quad Y = \frac{y - y_s}{b},$$
$$\theta = \frac{\tilde{T}}{Q/k}, \quad P = \frac{\tilde{p}}{\rho(\nu/b)^2}. \tag{10}$$

Note that Y denotes the dimensionless distance in a periodic module relative to the bottom boundary of the module. The scaling employed in equation (10) does not utilize the preferential scaling of v velocity and y dimension used by Aung [2], since the periodic fully developed flow is not one-dimensional. In reporting the results, the dimensionless flow rate is further scaled with respect to the Grashof number to exploit the functional dependence observed in a one-dimensional situation.

The Rayleigh number and the dimensionless length of the channel L (or scaled number of modules) are defined as follows:

$$Ra_b = \frac{g\beta b^3 Q}{\nu\alpha_f k_f} \quad \text{and} \quad L = \frac{\nu\alpha_f k_f l}{g\beta b^4 Q} \text{ with } l = Nl_m. \tag{11}$$

The dimensionless equations, then, have the following form:

Continuity

$$\nabla \cdot \bar{U} = 0. \tag{12}$$

Momentum equation

$$\bar{U}\cdot\bar{\nabla}\bar{U} = -\bar{\nabla}P + \bar{\nabla}^2\bar{U} + \frac{Ra_b}{Pr}\theta\hat{j} \tag{13}$$

where $\hat{j}$ denotes the unit vector in the vertical y direction.

Energy equation

Fluid:

$$\bar{U}\cdot\bar{\nabla}\theta = \frac{1}{Pr}\bar{\nabla}^2\theta - \frac{1}{Pr\,(l_m/b)}\frac{V}{\dot{M}}. \tag{14}$$

Block:

$$\bar{\nabla}^2\theta + \frac{1}{K(l_b/b)(b_b/b)} = 0. \tag{15}$$

The condition in equation (9) on the bulk temperature then becomes

$$\theta_{b,0} = \frac{1}{Pr}\frac{N}{2\dot{M}} \text{ or } \frac{LRa_b}{Pr\,(l_m/b)}\frac{1}{2\dot{M}} \tag{16}$$

where the dimensionless flow rate $\dot{M}$ is given by

$$\dot{M} = \int_0^1 V\,\mathrm{d}X. \tag{17}$$

The above equations are subject to the following boundary conditions:

Velocity field

$U = V = 0$ at $X = 0, 1$ and inside the block

U, V, P are periodic at $Y = 0$ and $Y = l_m/b$. (18)

Temperature field

$$\frac{\partial\theta}{\partial X} = 0 \text{ at } X = 0 \text{ and } 1$$

$$\theta \text{ is periodic at } Y = 0 \text{ and } Y = l_m/b. \tag{19}$$

The temperature field is, further, subject to the continuity of the heat flux at the boundary of the block and the fluid. This is expressed in dimensionless form as follows:

$$\left(\hat{n}\cdot\left(\bar{\nabla}\theta + \frac{1}{Pr\,(l_m/b)}\frac{1}{\dot{M}}\hat{j}\right)\right)_{\text{fluid}} = K\left(\hat{n}\cdot\left(\bar{\nabla}\theta + \frac{1}{Pr\,(l_m/b)}\frac{1}{\dot{M}}\hat{j}\right)\right)_{\text{block}} \tag{20}$$

where $\hat{n}$ denotes the normal to the interface between the block and the fluid.

It is of interest to interpret the condition specified by equation (16) in physical terms. The periodic temperature field (2) is subject to flux conditions on all the bounding surfaces so that equations (14) and (15) along with the boundary conditions in equation (19) can determine the temperature field only within an arbitrary constant. It is the length of the channel (or the number of modules) that determines the average level of the temperature in the channel and that is precisely the condition specified in equation (16). Further, the rise in the bulk temperature across the entire channel is exactly twice that of the level prescribed in equation (16). Thus, solving the above set of equations to obtain the periodically fully developed flow is equivalent to solving the flow in a module that is placed midway in the channel.

The parameters governing the periodically fully developed flow can be deduced easily from the dimensionless equations and boundary conditions. These are the Rayleigh number Ra_b and the length of the channel L (also interchangeable with N), which specify the operating conditions of the channel; the Prandtl number of the fluid, and the conductivity ratio K. In addition, the geometric parameters of the periodic module are the aspect ratio of the module $A_m = l_m/b$, the clearance $G = (b-b_b)/b$ between the block surface and the channel wall and the aspect ratio of the block $A_b = l_b/b_b$. It is noted that unlike the one-dimensional fully developed natural convection between parallel plates subjected to uniform heat flux studied by Aung [2], the Rayleigh number Ra_b, which denotes the heating rate in the module, is also a parameter apart from the dimensionless length of the channel L. This is due to the fact that the two-dimensionality in the module geometry results in the inertial forces, which are governed by heating rate per module, affecting the flow pattern.

COMPUTATIONAL METHOD

The equations governing the periodically fully developed flow are solved using a control volume based computational technique as described by Patankar [19]. In the discussion that follows, the discretization of the equations governing the flow field is described briefly followed by an explanation of the discretization of the temperature field, with special attention to the heat flux continuity condition at the interface. Finally, the overall solution procedure that addresses the condition on the level of the bulk temperature is described.

Discretization method

Grid layout. The domain of interest is divided into a set of control volumes. A main grid point is located at the center of each control volume. The functional variation of the quantity of interest is represented in terms of its values at those discrete points. The discretization equations for the values of a variable at a grid point are obtained by conserving the flux of this variable over the control volume that surrounds that grid point. The flux through a face is represented in terms of discrete values of the variable at the grid points that surround the face using the power law scheme proposed by Patankar [19]. In the presence of discontinuities in the material properties in the domain, as in the present study, the domain is divided into control volumes in such a manner that the discontinuities coincide with the control volume faces.

Flow field. An important feature of the discretization is the staggered mesh employed for locating the velocity components. For the rectangular grid in the present study, the u (v) velocities are displaced in the x (y) directions with respect to the main grid points that are used for the storage of pressure and other scalar variables. With this relative arrangement of nodal velocities and pressures, the occurrence of a checkerboard pressure field is avoided. The discretization equations for the velocities are obtained by writing momentum balances over the staggered control volumes. A detailed description of the discretization method is given by Patankar [19].

Temperature field. The discrete values of the periodic part of the temperature are stored at the main grid points and the discretization equations for these are obtained by conserving the total heat flux over the main control volumes. The decomposition of the temperature field into a linear part and a periodic part, as in equation (1), has interesting ramifications in its discretization when the domain has discontinuities in the thermal conductivity and is now discussed.

The heat flux at a point in the domain can be written in terms of dimensionless quantities as

$$\bar{q} = -(k\bar{\nabla}\tilde{T}+k\gamma\hat{j}). \qquad (21)$$

The first term on the right-hand side is the contribution to the heat flux due to the gradient of the periodic part of the temperature, while the second term is due to the superimposed linear variation. When the material conductivity is uniform across all faces of a control volume, the net contribution to the conduction heat flow into the control volume due to the linear variation of the temperature field is zero. Thus, only the periodic part of the temperature field needs to be considered in evaluating the conduction heat flux during the construction of the discretization equation for that control volume. However, when a face of a control volume coincides with an interface across which the thermal conductivity is discontinuous, the net contribution of the linear part of the temperature to the total conduction heat flow into the control volume is not zero. Further, for the special case of a rectangular grid oriented along the x and y coordinate directions, the discretization of the conduction heat flux should account for the linear part of the temperature only for those control volumes for which the discontinuity in thermal conductivity occurs across faces oriented along x-coordinate lines.

Consider such an interface, shown in Fig. 3, across which there is a discontinuity in the thermal conductivity. The total conduction heat flux q_y across a control volume face which lies along this interface is given by

$$q_y = \frac{k_{eq}\Delta x}{\Delta y}(T^- - T^+) \qquad (22)$$

where k_{eq} is the equivalent conductivity for the face and is obtained using the harmonic-mean approach of Patankar [19] as follows:

$$\frac{\Delta y}{k_{eq}} = \frac{\Delta y^+}{k^+} + \frac{\Delta y^-}{k^-}. \qquad (23)$$

Since the temperature difference $(T^- - T^+)$ can be expressed in terms of the periodic parts of the temperature field, q_y can be rewritten as follows:

$$q_y = \frac{k_{eq}\Delta x}{\Delta y}(\tilde{T}^- - \tilde{T}^+) + k_{eq}\Delta x(-\gamma). \qquad (24)$$

The first term in equation (24) is accounted for implicitly through the neighbor coefficients in the discretization equation for the periodic temperatures $\tilde{T}$. It is the second term that arises due to the linear variation that is responsible for additional source terms. These source terms for each of the two control volumes can be obtained by writing an expression similar to equation (24) for the heat flux through the other horizontal face of each of the two control volumes. By subtracting the contributions that arise due to the linear part of the temperature, the extra sources in the two control volumes can be written as follows:

$$S^+ = (-\gamma)\Delta x(k_{eq} - k^+)$$

and

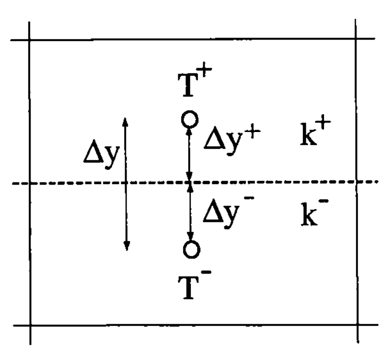

FIG. 3. An interface between control volumes with different thermal conductivities.

$$S^- = (-\gamma)\Delta x(k^- - k_{eq}). \quad (25)$$

It is noted that when $k_{eq} = k^+ = k^-$, these source terms vanish. In the problem considered, these source terms need to be computed for the control volumes bordering the top and bottom surfaces of the heated block. It is important to note that with this approach, the condition of continuity of heat flux across an interface is automatically satisfied and the entire periodic temperature field is treated in a unified fashion. The linear part of the temperature field also gives rise to a sink in the equation governing the periodic temperature field, in the fluid region, representing the heat absorbed by the convecting flow. Such a term is absent in the block since the block is stationary. The discretization of this term requires no special treatment.

Solution method

The velocity–pressure coupling is handled using the SIMPLER algorithm of Patankar [19]. Since the flow is driven by buoyancy forces, the velocity and temperature fields are coupled and the periodic part of the temperature field is solved along with the velocity field in every iteration. As discussed before, the level of the periodic part of the temperature cannot be determined from the discretization equations since the temperature field is subject to Neumann-type conditions on all boundaries. The level of the temperature is determined by the number of modules and the induced mass flow rate as expressed in the condition on the bulk value of this periodic temperature at the module entrance (equation (9)). The proposed solution method incorporates a procedure to ensure the satisfaction of this condition. Since the buoyancy force and hence the flow rate through the channel are, in turn, dependent on the level of the temperature, the mass flow rate is underrelaxed in order to provide numerical stability during the computations. The overall solution method is now outlined.

(1) Guess the field values of velocity, pressure and periodic part of the temperature and the corresponding throughflow $\dot{m}$.

(2) Solve the linearized form of the discretized momentum equations and the continuity equation using the SIMPLER algorithm of Patankar [19].

(3) Update the value of the mass flow rate using underrelaxation as follows:

$$\dot{m} = \alpha_m \dot{m}^{\text{new}} + (1-\alpha_m)\dot{m}^*. \quad (26)$$

Here, $\dot{m}^*$ and $\dot{m}^{\text{new}}$ denote the old and the new values of the mass flow rate, respectively, and α_m is the corresponding underrelaxation factor.

(4) Solve the algebraic equations for the grid point periodic temperatures.

(5) Compute the value of the periodic bulk temperature at the entrance of the module using

$$(\tilde{T}_{b,0})_{\text{computed}} = \frac{\sum |v_{ij}|\tilde{T}_{ij}\Delta x}{\sum |v_{ij}|\Delta x} \quad (27)$$

where the summation is carried out over the grid points on the periodic boundary of the module. The additive constant δ by which the periodic temperature field needs to be adjusted is computed as follows:

$$\delta = -\left[(\tilde{T}_{b,0})_{\text{computed}} - \frac{QN}{2\dot{m}c_p}\right]. \quad (28)$$

The entire temperature field is then incremented by δ as follows:

$$\tilde{T}_{ij} = \tilde{T}_{ij} + \delta. \quad (29)$$

(6) Repeat steps 2–5 until convergence.

COMPUTATIONAL DETAILS

In order to verify the correctness of the solution technique, the one-dimensional fully developed natural convection with heat flux on the left wall and insulated right wall is computed by taking a periodic module of an arbitrary aspect ratio (the value of which is immaterial) and imposing periodicity conditions on the top and bottom boundaries. The computed mass flow rate for a grid of 22 nodes across the channel is within 0.1% of the result predicted by Aung [2] for $L = 0.1$, 1 and 10. The solution procedure is then employed to compute the two-dimensional periodically fully developed natural convection in the channel geometry under consideration. Since the objective of the computations is to illustrate the utility of the proposed solution technique and to show the effect of two-dimensionality, the computations are carried out for a fixed geometry with a module aspect ratio $A_m = 3$, a gap width $G = 0.5$ and a block aspect ratio $A_b = 2$. The block is assumed to have a large conductivity relative to the fluid ($K = \infty$), as encountered in typical electronic cooling applications. The effect of the dimensionless channel length L on the mass flow rate is investigated by varying L between 10^{-2} and 10^2 for a fixed module Rayleigh number of $Ra_b = 1 \times 10^3$. In addition, the Rayleigh number Ra_b is varied between 1 and 5×10^4 for a fixed $L = 1$ to show the effect of two-dimensionality on the induced mass flow rate. All computations are carried out for $Pr = 0.7$. A 22×42 grid is used to discretize the periodic module. The grid is made fine near the solid surfaces in order to resolve the boundary layers. Computations on a grid twice as large in each direction show that the computed mass flow rates on the two grids differ by only 1% for the case of $Ra = 1 \times 10^3$ and $L = 10$. Therefore, the grid employed in the present study is deemed to be sufficiently fine to yield results of engineering accuracy.

The dimensionless stream function is normalized with respect to the module Grashof number as follows:

$$\psi = \frac{1}{Gr_b}\int_0^1 V\,dX. \quad (30)$$

In the results presented, the induced mass flow rate is also normalized by the module Grashof number. In order to quantify the resistance to heat transfer within a module, the module averaged Nusselt number is defined as

$$Nu = hb/k \quad (31)$$

where the heat transfer coefficient h is defined as

$$h = \frac{Q/(l_b + 2b_b)}{T_{block} - \bar{T}_b} \quad (32)$$

with T_{block} and $\bar{T}_b$ representing the uniform temperature of the block and the average of the bulk temperatures at the inlet and the exit module.

RESULTS AND DISCUSSION

An important outcome of the computations is the rate of the induced mass flow through the channel. The strength of the induced flow determines the level of the temperature that the heated components attain and hence is a measure of the effectiveness of cooling provided by natural convection. In addition, valuable insight into the underlying physical processes can be gained by examining the flow patterns and the temperature distributions that the computations reveal. The temperature varies continuously through the channel, with the rise in the bulk temperature across each module being the same. Therefore, in order to illustrate the details of the temperature variation, isotherms are plotted in a specific module. This module is located halfway through the channel so that the bulk temperature at the inlet of the module satisfies the condition specified by equation (2). The effect of the channel length and the module heating on the flow and the temperature field are now discussed.

Effect of channel length

For a specified geometry and module heating, as the channel length or the number of modules is increased, a longer column of heated air is created, giving rise to a larger buoyancy force. Figure 4 shows the streamlines and isotherms for three different module lengths. For small channel lengths, the induced flow is weak, so that the streamline pattern is similar to that of creeping flow. As the induced mass flow increases due to an increase in channel length, inertial effects become important and the flow separates. The temperature of the block is uniform due to the high conductivity of the block material and it is the highest temperature in the module. The effect of the increase in buoyancy force due to the increase in channel length is reflected in the larger induced mass flow rate through the channel, as shown in Fig. 5. It is of interest to note that because of the alteration of

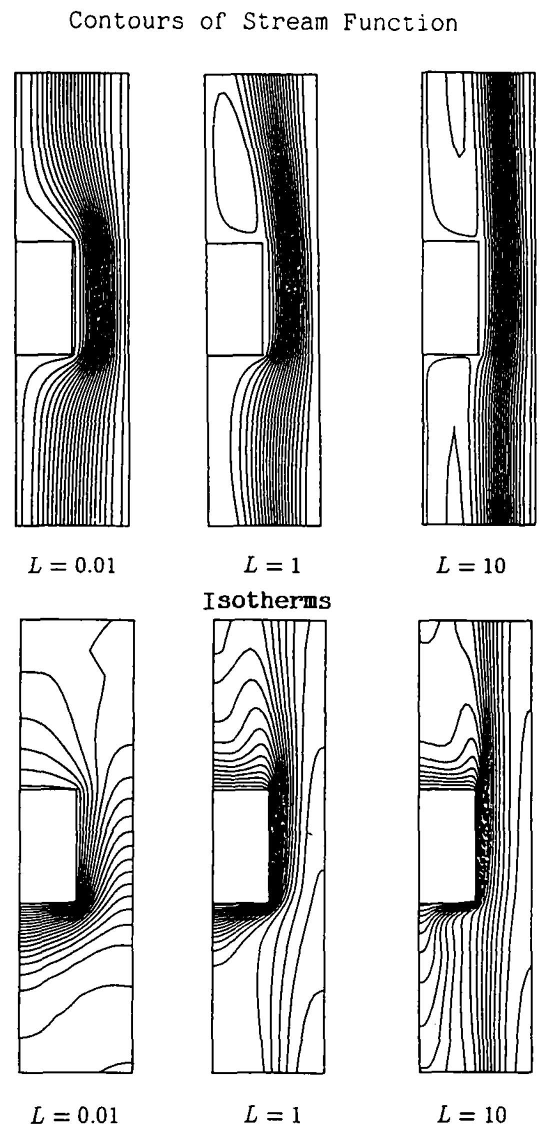

FIG. 4. Effect of channel length parameter on the flow and the temperature fields.

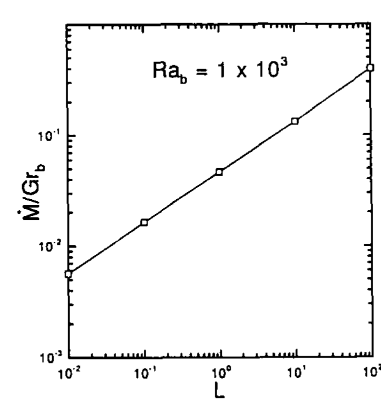

FIG. 5. Variation of the rate of the buoyancy-induced throughflow with the channel length parameter.

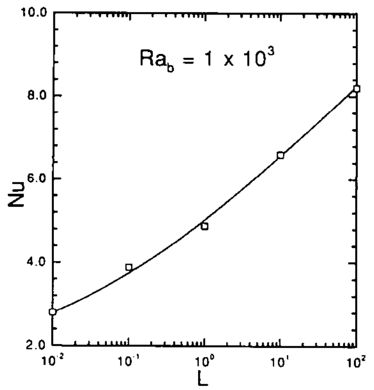

FIG. 6. Variation of the module averaged Nusselt number with the channel length parameter.

the flow resistance due to blockage, the slope of the curve giving the variation of the mass flow rate with channel length parameter is 0.46, which is less than 0.5, the figure corresponding to the fully developed flow through a parallel plate channel. Figure 6 shows the variation of the Nusselt number with the channel length parameter L. Note that the Nusselt number is a measure of the ease with which the heat is transferred from the block to the throughflow. As the strength of the throughflow increases, heat transfer from the block is high on the bottom as well as the side faces of the block. This is due to the washing effect of the throughflow causing the Nusselt number to increase with increasing channel length.

Some comments about the variation of the periodic part of the temperature over the module are in order. Figure 7 compares the variation of the temperature and the periodic part of the temperature over a module, the bulk value of both temperatures being the same at the module inlet. It should be noted that the static temperature is obtained by superimposing a linear part of the temperature on the periodic part. The periodic part of the temperature decreases linearly over the block so as to compensate for this superimposed linear variation, resulting in a uniform temperature of the block due to its high thermal conductivity.

Effect of module Rayleigh number

Figure 8 shows the effect of module Rayleigh number on the flow and the temperature field. The variations of the induced mass flow rate and the module averaged Nusselt number with the module Rayleigh number for a fixed value of the channel length parameter are shown in Figs. 9 and 10. At very low module heating, the throughflow is very weak so that the inertial effects are negligible, giving rise to a flow field and heat transfer pattern that are symmetric around the block. Under such conditions, the module Rayleigh number ceases to be a parameter and the

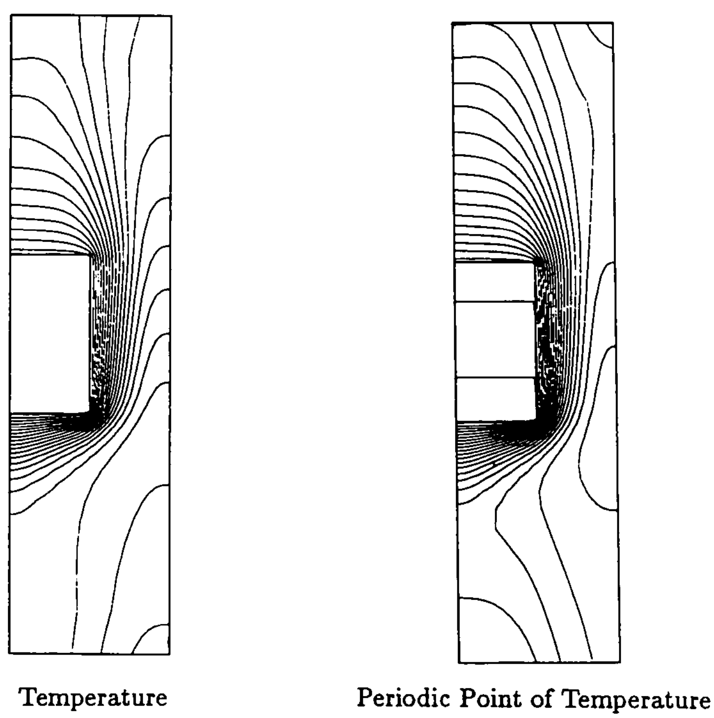

FIG. 7. Comparison of the temperature and the periodic part of the temperature in a module for $L = 0.1$ and $Ra_b = 10^3$.

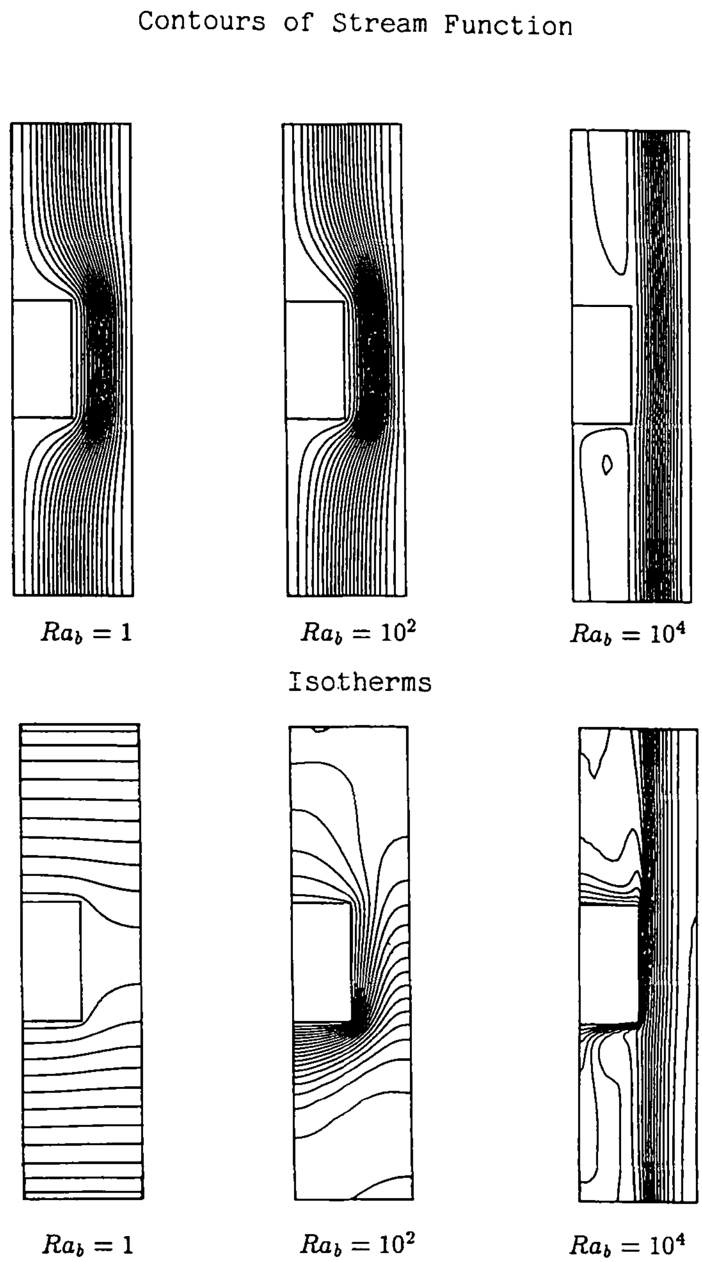

FIG. 8. Effect of module Rayleigh number on the flow and the temperature fields for $L = 1$.

flow and heat transfer characteristics are asymptotically independent of Ra_b, as seen from the plots in Figs. 9 and 10. However, as the module heating increases, the inertial effects become important and the flow characteristics depend on the module Rayleigh number. Furthermore, as the mass flow rate increases with increasing module heating, the resistance to the flow increases in more than direct proportion to the flow rate. Hence, the mass flow rate, when normalized with respect to the module Rayleigh number, decreases with increasing module Rayleigh number, as seen in Fig. 9. The module averaged Nusselt number increases with module Rayleigh number due to the decrease in the resistance to heat transfer from the block to the fluid. This is evident from the presence of thin boundary layers on the bottom and side surfaces of the block, as seen in Fig. 10.

CONCLUSIONS

Draft cooling is often utilized as a reliable mechanism for cooling of low powered electronic equip-

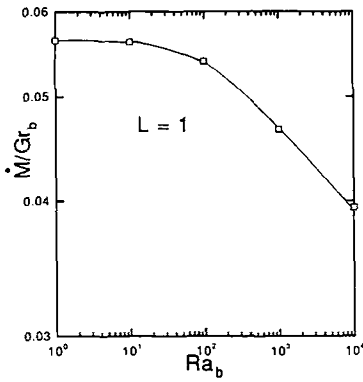

FIG. 9. Variation of the rate of the buoyancy-induced throughflow with the module Rayleigh number.

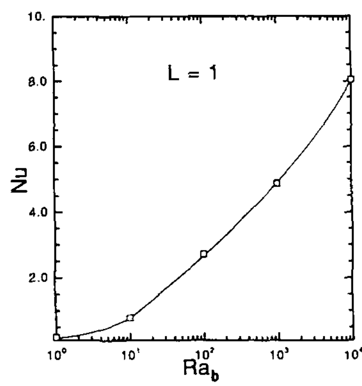

FIG. 10. Variation of the module averaged Nusselt number with the module Rayleigh number.

ment. In such applications, a periodically fully developed natural convection regime is attained after a short entry length, when the circuit boards are closely spaced. The effect of thermal boundary conditions on the nature of the periodically fully developed flow is discussed. A mathematical formulation is provided for the periodically fully developed natural convection flow, through a channel with insulated walls and heated blocks mounted on one of the walls. The temperature field is decomposed into a linear and a periodic part, which yields a decomposition of the pressure field into a parabolic and a periodic part. The discretization method is then outlined and the effect of the particular decomposition of the temperature on its discretization, in the presence of discontinuities in thermal conductivity in the domain, is detailed. A solution method, which accommodates the condition which prescribes the level of the periodic part of the temperature, is described. The mathematical formulation and the proposed solution procedure, although outlined in the context of a particular physical situation, are completely general and can be applied to analyze the type of periodically fully developed flows where there is absorption of heat by the throughflow. Computations are carried out for a fixed geometry of the periodic module in the channel to study the effect of the dimensionless length of the channel and the module Rayleigh number, which denotes the heating rate in the module. The results show that the mass flow rate increases at a rate less than the square root of the channel length. The effect of two-dimensionality on the induced mean flow rate, as evidenced in the variation of the induced throughflow rate with module heating, is not negligible. Such analyses can be utilized to perform parametric studies for design optimization of draft cooled electronic equipment.

REFERENCES

1. G. P. Peterson and A. Ortega, Thermal control of electronic equipment and devices, *Adv. Heat Transfer* **20**, 181–314 (1990).
2. W. Aung, Fully developed laminar free convection between vertical flat plates heated asymmetrically, *Int. J. Heat Mass Transfer* **15**, 1577–1580 (1972).
3. W. Aung, Heat transfer in electronic systems with emphasis on asymmetric heating, *Bell Syst. Tech. J.* **52**, 907–925 (1973).
4. W. Aung, L. S. Fletcher and V. Sernas, Developing laminar free convection between vertical flat plates with asymmetric heating, *Int. J. Heat Mass Transfer* **15**, 2293–2307 (1972).
5. W. Aung, T. J. Kessler and K. I. Beitin, Natural convection cooling of electronic cabinets containing arrays of vertical circuit cards, ASME Paper No. 72-WA/HT-40 (1972).
6. W. Aung, T. J. Kessler and K. I. Beitin, Free convection cooling of electronic systems, *IEEE Trans. Parts Hybrids Package* **PHP-9**, 75–86 (1973).
7. R. A. Wirtz and R. J. Stutzman, Experiments on free convection between vertical plates with symmetric heating, *J. Heat Transfer* **104**, 501–507 (1982).
8. E. M. Sparrow, G. M. Chrysler and L. F. Azevedo, Observed flow reversals and measured-predicted Nusselt numbers for natural convection in a one-sided heated vertical channel, *J. Heat Transfer* **106**, 325–332 (1984).
9. E. M. Sparrow and C. Prakash, Enhancement of natural convection heat transfer by a staggered array of discrete vertical plates, *J. Heat Transfer* **102**, 215–220 (1980).
10. C. Prakash and E. M. Sparrow, Natural convection heat transfer performance evaluations for discrete (in-line or staggered) and continuous plate arrays, *Numer. Heat Transfer* **3**, 89–105 (1980).
11. A. Bar-Cohen and W. M. Rohsenow, Thermally optimum spacing of vertical natural convection cooled, parallel plates, *J. Heat Transfer* **106**, 116–123 (1984).
12. H. Birnbreier, Experimental investigations on the temperature rise of printed circuit boards in open cabinets with natural ventilation. In *Heat Transfer in Electronic Equipment*, ASME HTD-20, pp. 19–23. ASME, New York (1981).
13. Y. Jaluria, Buoyancy-induced flow due to isolated thermal sources on a vertical surface, *J. Heat Transfer* **104**, 223–227 (1982).
14. Y. Jaluria and B. Gebhart, Buoyancy induced flow arising from a line thermal source on an adiabatic vertical surface, *Int. J. Heat Mass Transfer* **20**, 153–157 (1977).
15. L. F. Milanez and A. E. Bergles, Studies on natural convection heat transfer from thermal sources on a vertical surface, *Proc. 8th Int. Heat Transfer Conf.*, Vol. 3, pp. 1347–1352 (1986).
16. A. Ortega and R. J. Moffat, Heat transfer from an array

of simulated electronic components: experimental results for free convection with and without a shrouding wall. In *Heat Transfer in Electronic Equipment—1985*, ASME HTD-48, pp. 5–15. ASME, New York (1985).
17. A. Ortega and R. J. Moffat, Buoyancy-induced convection in a nonuniformly heated array of cubical elements on a vertical channel wall. In *Heat Transfer in Electronic Equipment—1986*, ASME HTD-57, pp. 135–144. ASME, New York (1986).
18. S. V. Patankar, C. H. Liu and E. M. Sparrow, Fully developed flow and heat transfer in ducts having streamwise-periodic variations of cross-sectional area. *J. Heat Transfer* **99,** 180–186 (1977).
19. S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*. McGraw-Hill, New York (1980).